

LINEAR PROGRAMMING PROBLEMS (LPP)

1. A factory makes tennis rackets and cricket bats. A tennis racket takes 1.5 hours of machine time and 3 hours of craftsman's time in its making while a cricket bat takes 3 hours of machine time and 1 hour of craftsman's time. In a day, the factory has the availability of not more than 42 hours of machine time and 24 hours of craftsman's time. (i) What number of rackets and bats must be made if the factory is to work at full capacity? (ii) If the profit on a racket and on a bat is Rs 20 and Rs 10 respectively, find the maximum profit of the factory when it works at full capacity.
2. A man rides his motorcycle at the speed of 50 km/hour. He has to spend Rs 2 per km on petrol. If he rides it at a faster speed of 80 km/hour, the petrol cost increases to Rs 3 per km. He has atmost Rs 120 to spend on petrol and one hour's time. He wishes to find the maximum distance that he can travel. Express this problem as a linear programming problem.
3. An aeroplane can carry a maximum load of 200 passengers. Baggage allowed to the first class ticket holder is 30 kg and for the economy class ticket holder is 20 kg. Maximum capacity of the aeroplane to carry the baggage is 4500 kg. The profit on each first class ticket is Rs.500 and on each economy class ticket is Rs.300. Formulate the problem, as L.P.P to maximize the profit.
4. A manufacturer has employed 5 skilled men and 10 semi-skilled men and makes two models A and B of an article. The making of one item of model A requires 2 h work by a skilled man and 2 h work by a semi-skilled man. One item of model B requires 1 h by a skilled man and 3 h by a semi-skilled man. No man is expected to work more than 8 h per day. The manufacturer profit on an item of model A is t 15 and on an items of model B is ? 10. How many of items of each models should be made per day in order to maximize daily profit? Formulate the above LPP and solve it graphically and find the maximum profit.
- 5 A factory manufactures two types of screws, A and B. Each type of screw requires the use of two machines, an automatic and a hand operated. It takes 4 minutes on the automatic and 6 minutes on hand operated machines to manufacture a package of screws A, while it takes 6 minutes on automatic and 3 minutes on the hand operated machines to manufacture a package of screws B. Each machine is available for at the most 4 hours on any day. The manufacturer can sell a package of screws A at a profit of Rs 7 and screws B at a profit of Rs10. Assuming that he can sell all the screws he manufactures, how many packages of each type should the factory owner produce in a day in order to maximize his profit? Determine the maximum profit.
6. There are two types of fertilisers A and B'. A' consists of 12% nitrogen and 5% phosphoric acid whereas B' consists of 4% nitrogen and 5% phosphoric acid. After testing the soil conditions, farmer finds that he needs at least 12 kg of nitrogen and 12 kg of phosphoric acid for his crops. If 'A' costs 110 per kg and B' costs ? 8 per kg , then graphically determine how much of each type of fertiliser should be used so that the nutrient requirements are met at a minimum cost?
7. A cottage industry manufactures pedestal lamps and wooden shades, each requiring the use of a grinding/cutting machine and a sprayer. It takes 2 h on the grinding/cutting machine and 3 h on the sprayer to manufacture a pedestal lamp. It takes 1 h on the grinding/cutting machine and 2 h on the sprayer to manufacture a shade. On any day, the sprayer is available for at the most 20 h and the grinding/cutting machine for at most 12 h. The profit from the sale of a lamp is ? 25 and that from a shade is ₹ 15. Assuming that the manufacturer can sell all the lamps and shades that he produces, how should he schedule his daily production in order to maximise his profit? Formulate an LPP and solve it graphically.
8. A dietician wishes to mix two types of foods in such a way that the vitamin contents of the mixture contains at least 8 units of vitamin A and 10 units of vitamin C. Food I contains 2 units per kg of vitamin A and 1 unit per kg of vitamin C. Food II contains 1 unit per kg of vitamin A and 2 units per kg of vitamin C. It costs ₹ 50 per kg to purchase food I and ₹ 70 per kg to purchase food II. Formulate the problem as a linear programming problem to minimise the cost of such mixture and find the minimise cost graphically.
9. A factory manufacturers two types of screws, A and B. Each type of screw requires the use of two machines, an automatic and a hand operated. It takes 4 minutes on the automatic and 6 minutes on hand operated machines to manufacture a package of screws A, while it takes 6 minutes on automatic and 3 minutes on the hand operated machines to manufacture a package of screws B. Each machine is available for at the most 4 hours on any day. The manufacturer can sell a package of screws A at a profit of Rs 7 and screws B at a profit of Rs 10. Assuming that he can sell all the screws he manufactures, how many packages of each type should the factory owner produce in a day in order to maximise his profit? Determine the maximum profit.
10. There are two types of fertilisers, F_1 and F_2 . F_1 consists of 10% nitrogen and 6% phosphoric acid, and F_2 consists of 5% nitrogen and 10% phosphoric acid. After testing the soil conditions, a farmer finds that she needs atleast 14 kg of nitrogen and 14 kg of phosphoric acid for her crop. If F_1 costs Rs 6/kg and F_2 costs Rs 5/kg, determine how much of each type of fertiliser should be used so that nutrient requirements are met at a minimum cost. What is the minimum cost?

PROBABILITY

- Two numbers are selected at random from the integers 1 through 9. If the sum is even, find the probability that both the numbers are odd.
- A die is rolled. If the outcome is an odd number, what is the probability that it is prime?
- A coin is tossed twice. If the outcome is at most one tail, what is the probability that both head and tail have appeared?
- An unbiased die is tossed twice. Find the probability of getting a 4, 5, 6 on the first toss and a 1, 2, 3, 4 on the second toss.
- The probability that person A hits a target is $\frac{1}{3}$ and the probability that person B hits it is $\frac{2}{5}$. What is the probability that the target will be hit if both person A and person B shoot at it?
- Find the binomial distribution of getting an even number if an unbiased dice is thrown thrice.
- How many times must a man toss an unbiased coin to get at least one head is more than 90%?
- Three cards are drawn with replacement from a well-shuffled deck of 52 cards. Find the probability distribution of the number of aces drawn. Also, find the mean and variance of the distribution.
- The probability of a trainee archer hitting the target is $\frac{1}{4}$. If he takes 7 shots, what is the probability of his hitting the target at least twice?

10. The probability distribution of a random variable X is given as under :

$$P(X = x) = \begin{cases} kx^2, & \text{for } x = 1, 2, 3 \\ 2kx, & \text{for } x = 4, 5, 6 \\ 0, & \text{otherwise} \end{cases} \quad \text{where } k \text{ is a constant}$$

Based on the above information , answer the following questions:

- (i) What is the value k? (ii) Find $P(X < 4)$ (iii) Find $E(X)$?

11. The probability distribution of a random variable X is given as under :

$$P(X = x) = \begin{cases} k, & \text{if } x = 0 \\ 2k, & \text{if } x = 1 \\ 3k, & \text{if } x = 2 \\ 0, & \text{otherwise} \end{cases} \quad \text{where } k \text{ is a constant}$$

Based on the above information , answer the following questions:

- (i) What is the value k? (ii) Find $P(X < 2)$ (iii) Find $P(X \leq 2)$

12. There are 4 cards numbered 1, 3, 5 and 7, one number on one card. Two cards are drawn at random without replacement. Let X denote the sum of the numbers on the two drawn cards. Find the mean and variance of X.

13. Two numbers are selected at random (without replacement) from positive integers 2, 3, 4, 5, 6 and 7. Let X denote the larger of the two numbers obtained. Find the mean and variance of the probability distribution of X.

14. From a lot of 15 bulbs which include 5 defectives, a sample of 2 bulbs is drawn at random (without replacement). Find the probability distribution of the number of defective bulbs

15. The mean and variance of the binomial variate X are 8 and 4 respectively. Find $P(X < 3)$.

16. The binomial variate X lies within the range $\{0, 1, 2, 3, 4, 5, 6\}$, provided that $P(X=2) = 4P(X=4)$. Find the parameter "p" of the binomial variate X.

17. In binomial distribution, X is a binomial variate with $n = 100$, $p = \frac{1}{3}$, and $P(X=r)$ is maximum. Find the value of r.

18. What is the mean of the numbers obtained on throwing a die having written 1 on three faces , 2 on two faces and 5 on one face ?

19. The probability that a person can achieve a target is $\frac{3}{4}$. The count of tries is 5. What is the probability that he will attain the target at least thrice?

20. A coin that is fair in nature is tossed n number of times. The probability of the occurrence of a head six times is the same as the probability that a head comes 8 times, then find the value of n.